

Loaded Rubber-Like Materials Subjected to Small-Amplitude Vibrations

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Abstract This paper proposes a constitutive law and a method for characterizing highly preloaded viscoelastic materials subjected to linear (small-amplitude) vibrations. A multiplicative non-separable variables law has been suggested to model the behavior that depends on both stretch and time/frequency. This approach allows splitting the intricate combined test performed simultaneously on both stretch and frequency, generally in a limited experimental domain up to 100 Hz, into two independent tests. Thus, on one hand, the dynamic complex modulus dependent on frequency alone is evaluated on the basis of vibration tests in a large experimental domain up to 100 kHz. On the other hand, energetic parameters are determined from a quasi-static hyperelastic tensile test. The complex modulus, dependent on both stretch and frequency, is then deduced from the results acquired from uncoupled investigations. This work shows that, in extension, the elastic modulus increases with increasing stretch, and the loss factor decreases with increasing stretch; while, in compression, around the material undeformed state, the modulus increases as the stretch increases till a certain value of compression stretch (upturn point depending on material characteristics), and then the modulus decreases as the stretch increases. Globally, preload rigidifies materials but reduces their damping property. These results closely match a well-known observation in solid mechanics.

Keywords: Elastomers; Viscoelasticity; Hyperelasticity; Vibrations; Modeling; Characterization; Combined modulus.

INTRODUCTION

Elastomers (such as rubbers, polyurethanes, fluorosilicons, foam rubbers, *etc.*) are widely used in industry for the suppression of noise and vibration, and calculations for structures using such materials have been well developed in recent years. Computer science (computers and digital software) has significantly contributed to this progress. Today, digital simulation has become competitive with the cost of developing prototypes. Elastomers can be compact, cellular and even composite. However, mechanical behavior is complex: these materials can be highly deformed by very little effort (instantaneously up to several hundreds percent). Internal forces are dependent on solicitation velocity and temperature. The thermomechanical behavior of such material should be perceived within the context of nonlinear viscoelasticity: geometrical nonlinearity and physical nonlinearity. Complex simulations can be achieved at present for quasi-static loads within the context of hyperelasticity. A certain number of models can be used to describe the quasi-static hyperelastic behavior of these materials while others focus on their viscoelastic behavior. The first point of view is that of simulation and calculation while the second concerns experimentation and identification. In viscoelasticity, two perspectives have emerged within the context of simulation:

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The first perspective is based on a mathematical approach and stipulates that the Piola-Kirchhoff stress tensor is linked to the strain tensor (Green-Lagrange) by multiple convolutions of the Volterra series type which involves kernel functions of any order. Lai and Findley^[1] limit themselves to symmetrical parabolic kernels of an order less than or equal to three, while Lockett^[2] proposes a method of obtaining kernels, and Molinari^[3] examines the one-dimensional problem. Huet^[4] treats the three-dimensional case of ageing materials. To the extent that other authors have generalized linear viscoelasticity tools, multiple convolutions are not easily exploitable from a practical perspective. This approach has a double advantage: it models simple convolution associated with known calibration system techniques.

The second approach was developed by Coleman and Noll^[5] who proposed a constitutive law (for materials with fading memory) developed in second order on the left Cauchy-Green tensor (current configuration)^[6,7]. The coefficients of development are simple convolutions expressed through the right Cauchy-Green strain tensor (reference configuration). This hybrid development favors the current configuration. Schapery model^[8,9] is close to linear viscoelasticity since it stipulates that total deformation is the sum of the instantaneous (non-linear) deformation and of the deformation resulting from memory effect. In this model, the nonlinearity of each term is expressed by multiplicative factors which depend on the variables considered, including time. Regarding the treatment of time, the factor of translation appears comparable to that of the "time-frequency" superposition in linear viscoelasticity (W.L.F master curves). This model, found in the works of Valanis and Landel^[6], was generalized by O'Dowd and Knauss^[10], who replaced the multiplicative products with functions of several variables. These different approaches have the common goal of treating nonlinearity *via* simple convolutions while neglecting imposed or possible discontinuities of functions.

The aim of this work is to propose a viscoelastic constitutive law for small deformations superposed on an initial large strain, while making use of simple convolution including hyperelasticity and possible functions discontinuity. Thus, one will try to construct a reliable model of small vibrations of viscoelastic solids under preload that are sufficiently rich and realistic to be introduced efficiently into computation code (calculation software). This model must be accessible for study by using separately classical techniques for hyperelastic quasi-static tests and for linear viscoelasticity experiments. It must also allow for extension of the experimental characterization domain that actually remains restricted up to 100 Hz around. Note that the general theory of linear vibrations of preloaded rubbery solids^[11] and interesting practical approaches and applications^[12-17] have previously been developed.

CONSTITUTIVE LAW FOR SUPERPOSED LINEAR VIBRATIONS ON PRELOADED VISCOELASTIC MATERIALS

In a large number of industrial applications, the material is subjected to a static preload $\mathcal{E}_0$ that is constant in time, around which are appended linear vibrations characterized by small deformations $\mathcal{E}(t)$. This is the case, for instance, of elastomer contact blocks of vibrant structures (turning machines, vehicle engine propeller units, transportation engines, *etc.*). There are two main approaches for modeling the behavior of a material subjected to a finite quasi-static deformation on which linear vibrations are superposed. The first approach focuses on the additive (sum) law which summarizes the independent effects of the predeformation and the frequency; and the second concerns the multiplicative (product) law that considers their mutual effects. See works of Govindjee and Reese^[18].

Additive Law

This approach is based on constitutive models linearization practice; equations are linearized in the neighborhood of a static pre-strain, so the entire deformation is a superposition of deformations. Interesting results have been obtained by this approach^[17, 19], even in software code implementations^[14, 20-22]. Considering the sum (additive) law, the deformation takes the following shape:

$$\mathcal{E}(t) = \mathcal{E}_0 + \mathcal{E}(t), \mathcal{E}(t) \ll 1 \quad (1)$$

$\mathcal{E}(t)$ is the strain tensor of Green-Lagrange; $\mathcal{E}_0(t)$ the static large deformation and $\varepsilon(t)$ the small (linear) deformation. The stress tensor of Piola-Kirchhoff takes the following form:

$$S(t) = S_0(t) + S_1(t) \quad (2)$$

where $S(t)$ is the dynamic stress, while $S_0(t)$ represents the static stress. The static stress S_0 tends to a constant value with time while the dynamic stress S_1 reflects the evolution around this constant stress. The material thus possesses a linear viscoelastic behavior (S_1, ε) around a state of permanent deformation ($S_0, \mathcal{E}_0$), with the relaxation function depending on permanent finite deformation. Morman and Nagtegaal^[14], Padovan^[15], Govindjee and Reese^[18] assume the superposition of pre-strain and frequency effects. The stress tensor is taken in the following form:

$$S^*(\mathcal{E}_0, \omega) = \Phi^*(\mathcal{E}_0) + \vartheta^*(\omega) \quad (3)$$

In this approach, it is understood that superposed linear vibration is not affected by the preload. The additive law remains well-adapted for evaluating stresses when the strains are known (large strains or linear strains) or vice versa. However, it is less well-adapted for determining the combined characteristics of a material.

Multiplicative Law

Generally, there exists inter-influence between effects of linear vibrations and preload, but the additive law does not take it into consideration. In this hypothesis, the multiplicative law is more adapted^[18]. For rubbery materials (for instance rubber containing fillers as carbon-black), it is known from experiments that the relaxation function depends on the static deformation^[16]. In the frequency domain, the complex dynamic stress is expressed as:

$$S_1^*(\omega) = R^*(\mathcal{E}_0, \omega) \varepsilon^*(\omega) \quad (4)$$

in which $R^*(\mathcal{E}_0, \omega)$ is the tensor of rank 4 of the complex tangent modulus depending on the deformation and on the frequency ω , i.e., the combined modulus. In the event of constant high deformation $\mathcal{E}_0$, the components of the tensor $R^*(\mathcal{E}_0, \omega)$ can be determined by classical linear viscoelasticity techniques, with all the difficulties posed by applying tension. Another way consists of using tangent modules defined by hyperelastic constitutive laws^[13]. Generally, tangent elastic modulus, with the deformation being $\mathcal{E}$, are defined by the following relation:

$$R(\mathcal{E}) = \frac{\partial(J\sigma)}{\partial F} F^T \text{ or } R_{pqkl}(\mathcal{E}) = \frac{\partial(J\sigma_{pq})}{\partial F_{ks}} F_{ls} \quad (5)$$

in which $F \equiv [F_{ij}]$ is the deformation gradient, J the determinant and σ the Cauchy stress tensor linked to the Piola-Kirchhoff stress tensor S via the relationship:

$$J\sigma = FST^T \quad (6)$$

S can be rewritten as:

$$S = 2 \frac{\partial W}{\partial C} \quad (7)$$

where C is the right Cauchy-Green strain tensor. Behavior modeling then consists of identifying the complex modulus tensor under high deformation $R^*(\mathcal{E}_0, \omega)$, Eq. (4), with the hyperelastic tangent tensor $R(\mathcal{E}_0)$, Eq. (5).

COMBINED COMPLEX MODULUS

To determine the combined complex modulus, we will base the study on two independent experiments: uniaxial tension test to evaluate material tangent modulus $E(\lambda)$ and linear uniaxial vibrations test to determine the dynamic complex modulus $E^*(\omega)$. Thus, we consider the multiplicative law and center our study on

incompressible materials under simple tension (linear viscoelasticity or hyperelasticity). These independent experiences and results will allow the evaluation of the complex modulus $E^*(\lambda, \omega)$ dependent on both frequency and stretch, *i.e.*, the combined complex modulus. Let us recall methods to evaluate $E^*(\omega)$ and $E(\lambda)$.

Dynamic Complex Modulus $E^*(\omega)$

Dynamic complex Young's modulus is determined at ambient temperature by the viscoanalyzer ISMCM (Institut Supérieur des Matériaux et de la Construction Mécanique) in a range of frequencies from about ten Hz to one MHz^[23–25]. The principle consists of, on the one hand, determining in extension-compression (or torsion) vibrations, the theoretical transfer function $H_{th}(\omega)$ of a crammed (filled) bar with a mass at each extremity, and on the other hand, the experimental transfer function $H_{ex}(\omega) = H(\omega)e^{i\theta(\omega)}$ from the Fourier analyzer (of the viscoanalyzer). Figures 1(a) and 1(c) show an experimental transfer function of the rubber sample referenced BX^[26]: respectively, gain and phase. Note that the theoretical transfer function $H_{th}(\omega)$ is expressed as^[24, 25]:

$$H_{th}(\omega) = \frac{1}{\cos \beta^* - \zeta \beta^* \sin \beta^*} \quad (8)$$

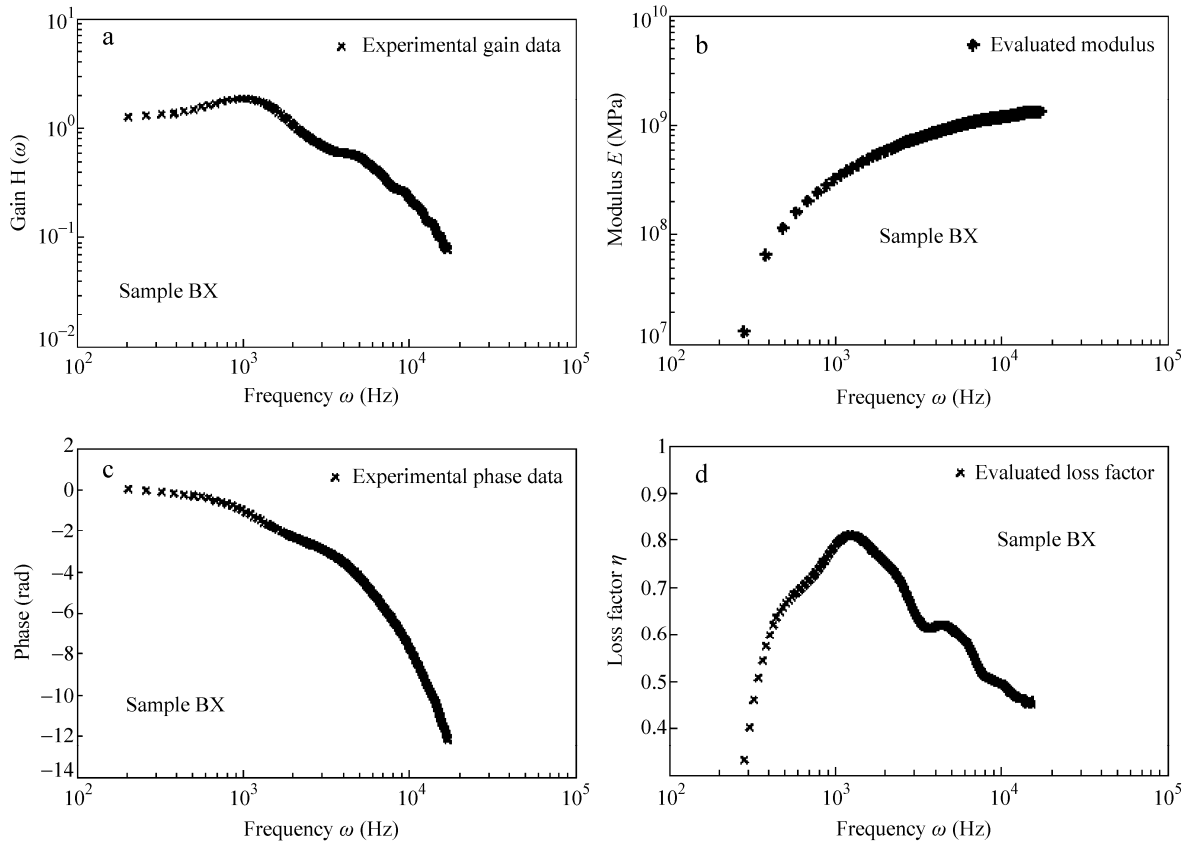


Fig. 1 (a) and (c): Experimental transfer function^[25]; (b) and (d): Evaluated complex modulus of the BX sample^[25]

where ζ is the ratio of the suspended masses, ρ the relative density and β^* is a complex variable given by:

$$\beta^* \equiv \beta^*(\omega) = L_0 \omega \sqrt{\frac{\rho}{E^*(\omega)}} \quad (9)$$

$E^*(\omega)$ is the sought-after complex dynamic modulus. By identifying the theoretical and experimental transfer functions: $H_{th}(\omega) = H_{ex}(\omega)$, *i.e.*:

$$\cos \beta^* - \zeta \beta^* \sin \beta^* = H^{-1} e^{-i\theta} \quad (10)$$

the complex dynamic modulus $E^*(\omega)$, that is, elasticity modulus and loss factor, is extracted from the transcendental function given by the Eq. (10), frequency by frequency, by nonlinear numerical methods. Let us write:

$$E^*(\omega) = E(\omega) e^{j\delta(\omega)} \quad (11)$$

$$= E'(\omega)(1 + j\eta(\omega)) \quad (12)$$

Figures 1(b) and 1(d) depict the complex Young modulus of the BX sample: its absolute value $E(\omega)$ and lost factor $\eta(\omega)$.

Tangent Modulus $E(\lambda)$

By considering the classical results, the tangent modulus $E(\lambda)$ is expressed in the following form:

$$E(\lambda) = \lambda \frac{\partial \sigma(\lambda)}{\partial \lambda} \quad (13)$$

and

$$\sigma(\lambda) = \frac{\partial W(\lambda)}{\partial \lambda} \quad (14)$$

where λ is the stretch ($\lambda = \lambda_1 = 1/\lambda_2^2 = 1/\lambda_3^2$, λ_j principal elongations). This can be rewritten, when expressing the energy function in terms of the two first strain-invariants (I_1 and I_2 , $I_3 = 1$ as the material is incompressible) of the Cauchy-Green right tensor C :

$$\sigma(\lambda) = 2 \left(\lambda^2 - \frac{1}{\lambda} \right) \left(\frac{\partial W}{\partial I_1} + \frac{1}{\lambda} \frac{\partial W}{\partial I_2} \right) \quad (15)$$

with

$$I_1 = \lambda^2 + \frac{2}{\lambda} \quad (16)$$

and

$$I_2 = 2\lambda + \frac{1}{\lambda^2} \quad (17)$$

Beda^[27] has reviewed several models for analyzing the rubber behavior for different modes of deformation (compression, extension, shear, equibiaxial): the constrained models of Gent-Thomas and Herman-Mannoré, the complete models of Boyce-Arruda, Yeoh-Fleming, Ogden^[28], Pucci-Saccomandi^[29], Beda-2005^[30] and Beda-2007^[27]. By comparing these models with one another and with experimental data, it was shown by using the Mooney plot analysis that for phenomenological models: those of Ogden, Pucci-Saccomandi, and Beda-2007 provide the best description of the stress-stretch response of any mode of deformation, and are well-adapted for Code implementation^[31]. For this study, we consider the earlier models of Mooney-Rivlin and Gent-Thomas, and the Ogden model. The classical Mooney-Rivlin model is expressed as:

$$W = A(I_1 - 3) + K(I_2 - 3) \quad (18)$$

That of Gent-Thomas is given by:

$$W = A(I_1 - 3) + K \ln \frac{I_2}{3} \quad (19)$$

A and K are material characteristics. From an experimental tension curve, the characteristics of hyperelastic models can be evaluated^[32]. Table 1 presents the estimated characteristics values of some classical materials (Treloar rubber^[33], Yeoh-Fleming vulcanizate^[34]). The models (cited above) predictions depict suitably

experimental data in a domain ranging from relatively small to medium deformation (transition zone). Good agreement between experimental acquisitions and models predictions is obtained.

Table 1. Estimated values^[27] of material characteristics for different models^[27]

Material	Model	Parameter A	Parameter K
Treloar rubber ^[33]	Mooney-Rivlin	0.092 MPa	0.145 MPa
	Gent-Thomas	0.127 MPa	0.300 MPa
Yeoh vulcanizate ^[34]	Mooney-Rivlin	0.170 MPa	0.121 MPa
	Gent-Thomas	0.127 MPa	0.283 MPa

In an invariant-based model, *i.e.*, $W \equiv W(I_1, I_2)$, the Eq. (13) can be rewritten as:

$$E(\lambda) = 2 \left(2\lambda^2 + \frac{1}{\lambda} \right) \frac{\partial W}{\partial I_1} + 2 \left(\lambda + \frac{2}{\lambda^2} \right) \frac{\partial W}{\partial I_2} \quad (20)$$

This relation can be more explicit by the choice of the energy model. Consider:

$$E(\lambda) = [A g_1(\lambda) + K g_2(\lambda)] \quad (21)$$

The functions $g_i(\lambda)$ are defined for the:

—Mooney-Rivlin model by:

$$g_1(\lambda) = 2 \left(2\lambda^2 + \frac{1}{\lambda} \right); \quad g_2(\lambda) = 2 \left(\lambda + \frac{2}{\lambda^2} \right) \quad (22)$$

—Gent-Thomas model by:

$$g_1(\lambda) = 2 \left(2\lambda^2 + \frac{1}{\lambda} \right); \quad g_2(\lambda) = 2 \frac{(2 + \lambda^3)}{(1 + 2\lambda^3)} \quad (23)$$

Combined Complex Modulus $E^*(\lambda, \omega)$

To consider the simultaneous effects of the static strain on the relaxation function, Kim and Youn^[16] introduce a notion of static deformation correction factor on the linearized Simo viscoelastic model^[35], in which the separation of variables frequency/deformation is assumed. In this approach, the complex Young's modulus takes the following form:

$$E^*(\lambda, \omega) = c^*(\lambda) g^*(\omega) \quad (24)$$

$c^*(\lambda)$ is the static deformation correction factor. Let us note that Cho and Youn^[36] considered the (superposed) dynamic strain amplitudes besides static deformation and frequency. The complex Young's modulus is taken as:

$$E^*(\lambda, \omega, \varepsilon) = c^*(\lambda) g^*(\omega) a^*(\varepsilon) \quad (25)$$

where $a^*(\varepsilon)$ is the dynamic amplitude influence factor. These models are adapted for characterizing materials (as filled rubbers), when determining the combined elasticity modulus and the combined loss factor^[16, 22, 36].

Among possible identification procedures, including those cited before, we choose that which can be resumed according to the concepts of linear viscoelasticity (correspondence principle), which can be resumed as to substitute the real parameters of the hyperelastic model by complex parameters depending on the frequency ω . These complex parameters must be evaluated by dynamic tests adapted to the constitutive law chosen^[13]. In this work, it is intended to apply this approach to an incompressible material governed by a hyperelastic law (homogenous and isotropic materials, Green). The obvious interest of this modeling is to enable characterizing the behavior of linear vibrations superposed on largely preloaded material by basing the approach on two independent classical tests: a linear vibration test performed on a material^[23, 24] and a classical uniaxial tension test in large deformation on the same material.

According to the correspondence principle, the passage to the viscoelastic system is done in agreement with Eqs. (4) and (21):

$$E^*(\lambda, \omega) = [A^*(\omega)g_1(\lambda) + K^*(\omega)g_2(\lambda)] \quad (26)$$

It is assumed the following additional hypothesis defining the multiplicative model:

$$A^*(\omega) = A g^*(\omega) \quad (27)$$

$$K^*(\omega) = K h^*(\omega) \quad (28)$$

in which $g^*(\omega)$ and $h^*(\omega)$ are two functions of complex values characterizing the material. At low frequencies ($\omega \equiv 0$), this hypothesis must lead to the tangent modulus $E(\lambda)$, Eq. (21), for any stretch λ . The functions g^* and h^* should then verify the following boundary conditions:

$$g^*(0) = h^*(0) = 1 \quad (29)$$

The conditions are fulfilled in particular if:

$$g^*(\omega) = \psi(\omega) \quad (30)$$

and

$$h^*(\omega) = \psi(\omega)[1 + i\xi(\omega)] \quad (31)$$

on condition that:

$$\begin{cases} \psi(0) = 1 \\ \xi(0) = 0 \end{cases} \quad (32)$$

According to the hypotheses of Ferry^[37], $\psi(\omega)$ can be assumed as a real function, thus Eq. (26) can be rewritten as:

$$E^*(\lambda, \omega) = E'(\lambda, \omega)[1 + j\eta(\lambda, \omega)] \quad (33)$$

$E'(\lambda, \omega)$ is the combined storage modulus and $\eta(\lambda, \omega)$ the combined loss factor. They respectively take the following form:

$$E'(\lambda, \omega) = [A g_1(\lambda) + K g_2(\lambda)] \psi(\omega) \quad (34)$$

$$\eta(\lambda, \omega) = \frac{K g_2(\lambda) \xi(\omega) \psi(\omega)}{A g_1(\lambda) + K g_2(\lambda)} \quad (35)$$

The functions $\psi(\omega)$ and $\xi(\omega)$ are determined from the knowledge of the linear viscoelastic characteristics of the material ($E'(\omega)$ and $\eta(\omega)$ without preload). In fact, $g_1(1) = 6\nu$ and $g_2(1) = 2\theta$, where parameters ν and θ are equal respectively to one ($\nu = 1$) and three ($\theta = 3$) for the Mooney-Rivlin model; to one ($\nu = 1$) and one ($\theta = 1$) for the Gent-Thomas model; and to a quarter ($\nu = 0.25$) and to three quarters ($\theta = 0.75$) for the Ogden model. Hence according to Eqs. (34) and (35):

$$E'(\omega) = \lim_{\lambda \rightarrow 1} E'(\lambda, \omega) = E'(1, \omega) = (6\nu A + 2\theta K) \psi(\omega) \quad (36)$$

$$\eta(\omega) = \lim_{\lambda \rightarrow 1} \eta(\lambda, \omega) = \eta(1, \omega) = \frac{2\theta K}{(6\nu A + 2\theta K)} \xi(\omega) \psi(\omega) \quad (37)$$

Note that the static Young's modulus is given by: $E_0 = E(1, 0) = 6\nu A + 2\theta K$. Finally the combined complex modulus for linear vibrations superposed on highly preloaded material is thus determined and expressed by the following relations:

$$E'(\lambda, \omega) = \frac{A g_1(\lambda) + K g_2(\lambda)}{6\nu A + 2\theta K} E'(\omega) \quad (38)$$

$$\eta(\lambda, \omega) = \frac{(6\nu A + 2\theta K) g_2(\lambda)}{2\theta [A g_1(\lambda) + K g_2(\lambda)]} \eta(\omega) \quad (39)$$

or under an equivalent form:

$$E'(\lambda, \omega) = \alpha_{AK}(\lambda) E'(\omega) \quad \text{Real part } (\equiv E_r(\lambda, \omega)) \quad (40)$$

$$E''(\lambda, \omega) = \beta(\lambda) E''(\omega) \quad \text{Imaginary part } (\equiv E_i(\lambda, \omega)) \quad (41)$$

$$\eta(\lambda, \omega) = \gamma_{AK}(\lambda) \eta(\omega) \quad \text{Loss factor} \quad (42)$$

with:

$$\alpha_{AK}(\lambda) = \frac{A g_1(\lambda) + K g_2(\lambda)}{(6\nu A + 2\theta K)} \quad (43)$$

$$\beta(\lambda) = \frac{g_2(\lambda)}{2\theta} \quad (44)$$

$$\gamma_{AK}(\lambda) = \frac{g_2(\lambda)}{2\theta \alpha_{AK}(\lambda)} \quad (45)$$

$$E''(\omega) = E'(\omega) \eta(\omega) \quad (46)$$

The complex dynamic Young's modulus can take the following form:

$$E^*(\omega) = |E^*(\omega)| \cos \delta(\omega) [1 + i \tan \delta(\omega)] \quad (47)$$

$$E^*(\omega) = E(\omega) \cos \delta(\omega) [1 + i \tan \delta(\omega)] \quad (48)$$

So,

$$E'(\omega) = E(\omega) \cos \delta(\omega) \quad (49)$$

and

$$\tan \delta(\omega) = \eta(\omega) \quad (50)$$

where $\delta(\omega)$ is the loss angle.

Nowadays the adapted expression (continuous function) for the complex dynamic modulus is the fractional model^[38] given by:

$$E^*(\omega) = \frac{z_1 + z_2 (i\omega)^\alpha}{1 + c (i\omega)^\alpha} \quad (51)$$

where c and α are real numbers, z_1 and z_2 complex numbers. The parameters of the fractional model are more simply to identify. They are estimated from discrete function of complex dynamic modulus evaluated previously, for example by using the [6.23 dB, 7.19 dB] technique, *i.e.*, Tibi diagram^[39–41].

RESULTS AND ANALYSIS

Analysis of the Weighting Functions $\alpha_{AK}(\lambda)$ and $\gamma_{AK}(\lambda)$

From Eqs. (40) and (42), it can be seen that the combined storage modulus $E'(\lambda, \omega)$ is obtained by weighting the dynamic elasticity modulus without preload $E'(\omega)$ by the function $\alpha_{AK}(\lambda)$ (which can be called static deformation correction factor on dynamic storage modulus and depends on material hyperelastic parameters). The combined loss factor is obtained by weighting the dynamic loss factor without preload $\eta(\omega)$ by the function $\gamma_{AK}(\lambda)$, *i.e.*, a static deformation correction factor on dynamic loss factor. Thus, the examination of the weighting-function

$\alpha_{AK}(\lambda)$ allows predicting the behavior of the combined modulus $E'(\lambda, \omega)$ from $E'(\omega)$; while the analysis of $\gamma_{AK}(\lambda)$ allows predicting that of the combined loss factor $\eta(\lambda, \omega)$ from $\eta(\omega)$. Both weighting-functions $\alpha_{AK}(\lambda)$ and $\gamma_{AK}(\lambda)$ depend on the material characteristics. Let us note that for the model developed in this paper, the relaxation function is under a non-separable variables form while the combined storage modulus and loss factor are under separable variables form.

From analysis of the Figs. 2(a) and 2(b) concerning Treloar^[33] and Yeoh data^[34], it is seen that for the Mooney, Gent-Thomas, the function $\alpha_{AK}(\lambda)$ increases with increasing λ in extension and around the material undeformed state in compression; but beyond a certain value of compression-stretch, *i.e.*, λ_u (upturn point) depending on material characteristics, it decreases as the stretch increases. Thus, one can say that for $\lambda > \lambda_u$ the combined storage modulus $E'(\lambda, \omega)$ increases with increasing stretch λ , and for $\lambda < \lambda_u$ it decreases with increasing λ . Let us remark that the upturn point does not occur at the undeformed state, thus, $\alpha_{AK}(\lambda)$ is not a symmetric function about $\lambda = 1$ (undeformed state). According to Voet and Morawski^[42], elucidating the phenomenon, the initial decrease in compression curve is due to the disruption and the subsequent increase at higher deformation is attributed to the limited extensibility of the elastomer chain. Kim and Yoon^[16] have tested three kinds of rubber with different carbon black contents. From these experiments, they found the same results, showing that depending on the contents, the storage modulus of the filled rubber shows an initial decrease followed by an increase with increasing static strain in compression.

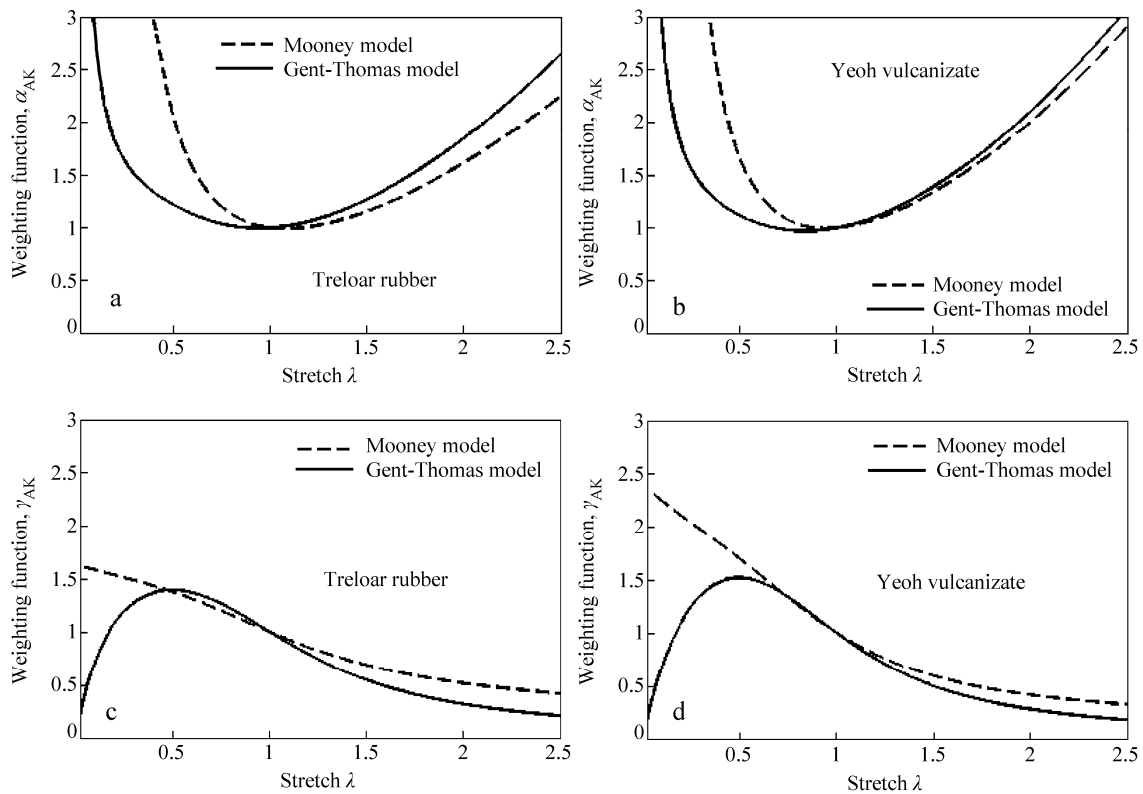


Fig. 2 Different models for different materials: (a) and (b) weighting function for modulus; (c) and (d) weighting function for loss factor

But on the other hand, the analysis of the weighting-function $\gamma_{AK}(\lambda)$ and from Figs. 2(c) and 2(d), it sets out that for the Mooney model $\gamma_{AK}(\lambda)$ decreases with increasing λ , while for the Gent-Thomas model, the combined loss factor becomes smaller as the value of compression-stretch becomes lesser, *i.e.*, λ tending to zero. So, the Gent-Thomas model prediction of the loss factor presents a downturn point in compression, Fig. 2.

Results

For this study, the Gent-Thomas constitutive law is chosen as it seems to be a more realistic model by its rational predictions of the loss factor and storage modulus. Furthermore, it is a very attractive model for small to moderate deformations since it is suitable for any deformation mode prediction. Also, it enables the material characterization by a single type of test, namely the uniaxial tensile test^[27]. Figures 3(a) and 3(c) represent the experimental tension curves and Figs. 3(b) and 3(d) the predictions of the Gent-Thomas model respectively for the BX sample and for the Kim-Youn NR/CB0 (natural rubber) material. The hyperelastic characterization is performed on the basis of the reduced stress curve by using the multi-stages approximation procedure^[32, 43], which is a continuous approximation method by blocks. Table 2 presents the values of the BX and Kim-Youn parameters materials.

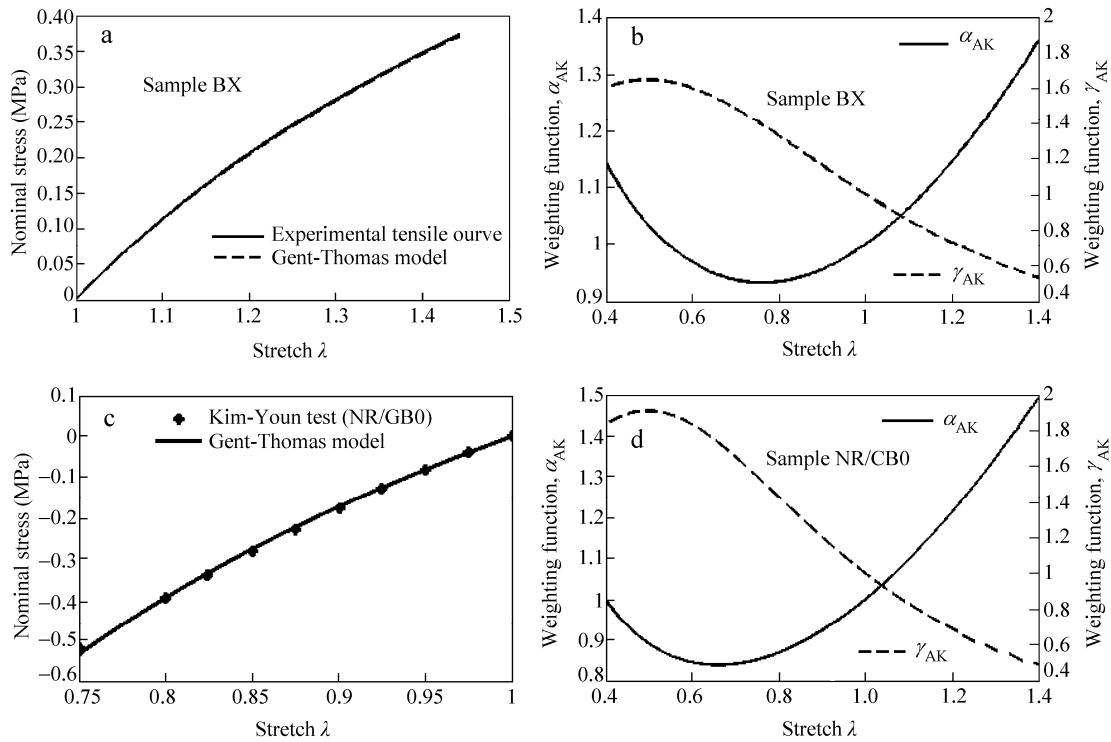


Fig. 3 Tension and weighting functions for the BX^[26] and the Kim-Youn NR/CB0^[16] samples: (a) and (c): Experimental tension curves, (b) and (d): Evaluated weighting functions

Table 2. Evaluated parameter values of the Gent-Thomas model

Material	Parameter A	Parameter K
BX sample	0.1630 MPa	0.145 MPa
Kim NR/CB0	0.2403 MPa	0.050 MPa

Let us note that the theoretical upturn point of the weighting-function $\alpha_{AK}(\lambda)$ occurs at $\lambda_u = 0.7603$ for the BX sample^[26] and at $\lambda_u = 0.6586$ for the Kim-Youn NR/CB0^[16]. The model developed here enables establishing dynamic behavior as a double variables function. Figures 4(a) and 4(b) show predicted combined complex modulus of BX sample for different values of stretch in a large range of frequencies from about 20 Hz to 100 kHz. Figures 4(c) and 4(d) show the storage modulus and loss factor as continuous double variables function of stretch and frequency. Figure 5(a) depicts the storage modulus of the natural rubber NR/CB0 from Kim-Youn tests versus frequency for various extensions. Figure 5(b) shows the Kim-Youn data in comparison with the model predictions deduced from the experimental master curve of the undeformed material state ($\lambda = 1$).

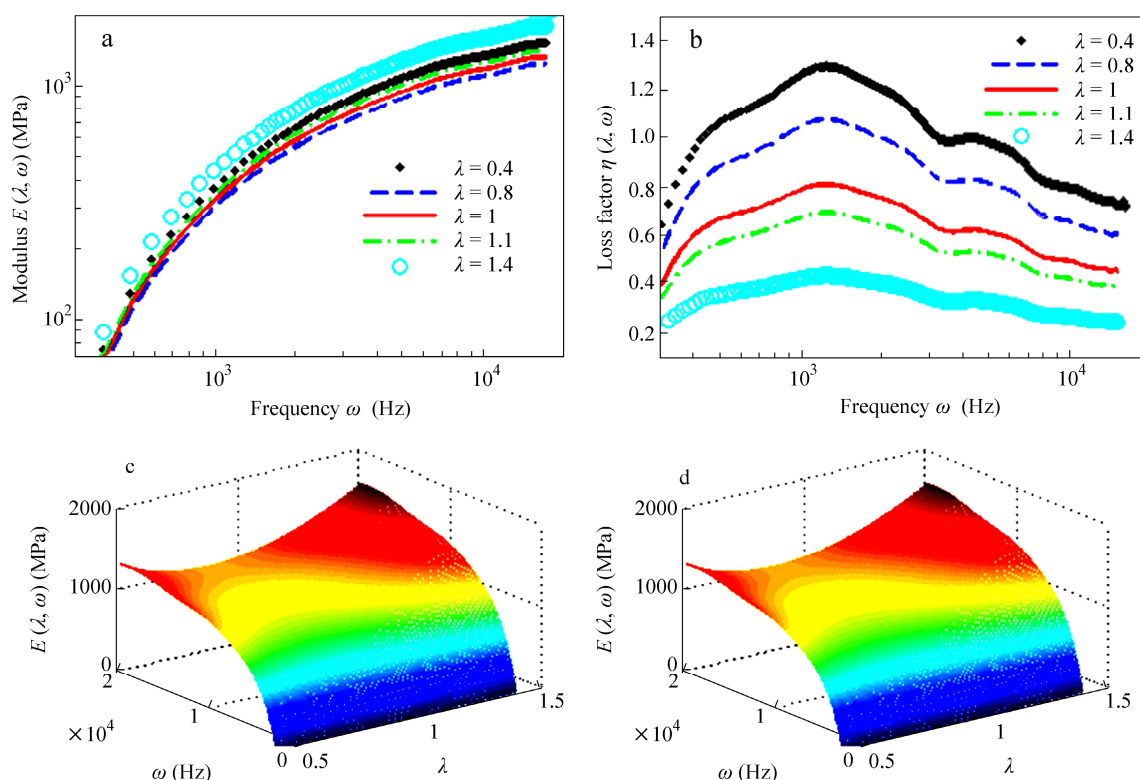


Fig. 4 Predictions of combined complex modulus of the BX sample versus frequency and extension: (a) and (c) combined elasticity modulus; (b) and (d) combined loss factor

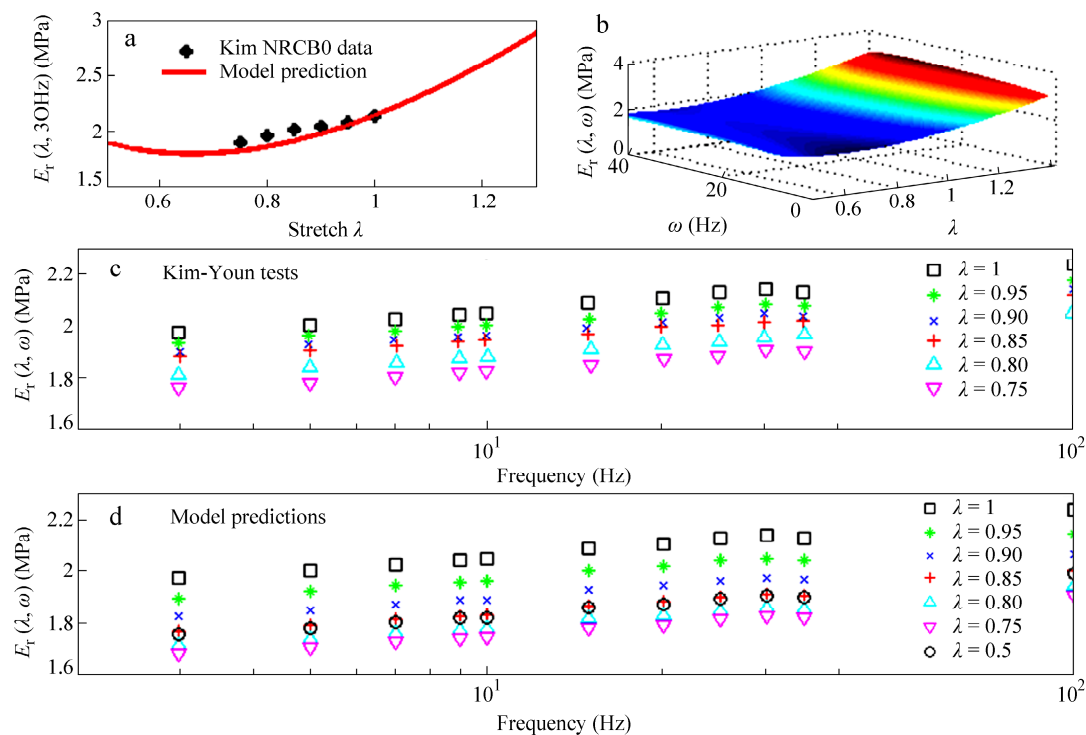


Fig. 5 Complex combined modulus of the Kim-Youn NR/CB0 material^[16]: (a) and (b) Data and predictions of storage modulus versus stretch at 30 Hz; (c): Kim-Youn tests^[16], (d) Predictions from the Gent-Thomas model

These results illustrate the following phenomenon: a preload (tension or compression) globally rigidifies a material while it reduces its damping property. Note that compression first makes it supple till the upturn point, and then rigidifies it.

CONCLUSIONS

The main interest of this work is to show that it is possible to characterize the dynamic behavior of a preloaded viscoelastic material by separating static tests under large deformations from linear vibration tests without preload. A non-separable variables model has been proposed for the relaxation function of nonlinear viscoelasticity. This approach greatly extends the material characterization domain up to 100 kHz, as the frequency range of traction machines under preload remains very restricted (below 1 kHz) and decreases as much as the load increases. This work shows the fact that the effects of the stretch on the relaxation function present a dual behavior in compression and tension. In other words, tension rigidifies a material but compression first makes it supple and then rigidifies it.

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